Promoting rich mathematical talk in geometry lessons: a concept taken up quickly by teachers but difficult to implement during lesson study

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Abstract

This study aimed at exploring whether and how learners' tasks planned by teachers and their implementation promoted learner participation in rich mathematical talk during lesson study on Pythagoras Theorem. Data included two lesson plans and two lessons planned and delivered by a group of five teachers from one secondary school in Malawi. Each lesson was video recorded, transcribed and divided into lesson episodes for content analysis using codes developed from the Mathematics Teaching Framework. The findings indicated that the teachers were able to develop tasks which could support learners' participation in rich mathematical talk. However, realisation of such rich mathematical talk for learners was a challenge during task implementation. I conclude that promotion of rich mathematical talk among learners is difficult and cannot be achieved through one lesson study cycle. This calls for the need for regular continuous professional development to promote in-service teachers' capacity to engage learners in rich mathematical talk as well as enriching pre-service mathematics education content with such opportunities.

Key Words: *Lesson study, rich mathematical talk, Mathematics Teaching Framework, right-angled triangle*

Introduction

Secondary school geometry teaching and learning has been recognised as challenging in Malawi and there are calls for mathematics teacher professional development (PD) to enhance teaching quality (Ministry of Education, Science and Technology [MoEST], 2020). In responding to this call, we initiated a Lesson Study (LS) project focused on geometry with 10 Malawian secondary mathematics teachers from two schools from 2021. After originating from Japan over 100 years ago, LS has been widely taken up in many countries as an effective PD for improving teacher knowledge and practice (Huang et al., 2019). However, uptake of LS is relatively new in Malawi and mainly focused at elementary mathematics (Fauskanger et al., 2019).

Research recommends the use of theory-informed approaches when introducing LS in a new context so as to structure teachers' discussion and learning, and enable carrying out systematic research of teacher learning (Huang et al., 2019). We therefore used the Mathematics Teaching Framework (MTF) a social cultural theoretically informed framework used in LS in a similar South African teaching and learning context to Malawi to structure teachers' discussions during the LS (Adler & Alshwaikh, 2019). MTF views mathematics as a network of scientific concepts and its teaching and learning as goal directed and mediated. It comprises three recognisable and inter-connected teaching practices: exemplification (examples, tasks and representations), explanatory communication (word use and justifications) and learner participation (doing and talking mathematics).

We conducted two cycles of LS with the teachers. In each cycle, we began the LS by conducting a two-day PD workshop with the secondary mathematics teachers. In the PD of the first cycle, we (1) discussed with the teachers activities for promoting geometric reasoning using the three teaching practices of MTF, and (2) introduced the teachers to LS mode of PD and its stages. Upon studying teachers' learning through participation in the first LS cycle, we confirmed that theory-informed LS supports adapting LS in a new context and reported on how LS can support teachers' learning of exemplification teaching practice within the MTF (Mwadzaangati et al., 2022; 2024). However, we observed that there was little attention to explanatory communication, a connected key teaching practice in the MTF that focuses on language. Thus, both the teachers' and learners' talk were mainly characterised by partial elaboration of several geometrical concepts, thus learner participation was limited to reporting measurements but not expression of their thinking on such measurements (Adler et al., 2022). We concluded from these observations that the teachers' involvement of learners in rich mathematical talk, also called language responsive teaching, was a challenge. Rich mathematical talk involves moving conversations from simply naming mathematical concepts and procedures towards making meaningful and purposeful elaborations through explaining and justifying the mathematical concepts (Adler & Alshwaikh, 2019; Prediger, 2022). Considering that learners' mathematical conceptual development is developed through their participation in rich mathematical talk (Barwell, 2018; Prediger, 2022), then the teachers' language practice challenges became a concern for action.

In cycle 2, we aimed at addressing the teachers' language challenges by focusing on explanatory communication and learner participation teaching practices of MTF. Our focus was to help the teachers to plan, enact and reflect on how to use words to communicate geometric concepts, and how to engage learners in expressing their geometric thinking. In mathematics education, these language practices are associated with rich mathematical talk or language responsive teaching (Prediger, 2022). After the 2 days-workshop, one group of the secondary school teachers decided to do a LS on how they could improve the teaching and learning of Pythagoras Theorem (PT), a topic in the lower secondary curriculum in Malawi. Their observation was that learners experience challenges in recognising the hypotenuse and that they sometimes apply PT in triangles that are not right-angled.

PT is one of the main foundations in mathematics problem solving (Veloo et al., 2015). There is a direct link between understanding of PT and other geometric concepts such that learners' strong understanding of PT leads to their strong understanding during learning other geometry concepts (Suryadi & Rosjanuardi, 2020). From research, the learners' challenges in PT include inability to: 1) understand the PT formula used in mathematics problem solving, 2) to solve problems involving PT according to the questions and, 3) to associate PT with trigonometry, and 4) understanding of the PT definition, notations, applications, and proofs (Suryadi & Sojanuardi, 2020). The challenges of understanding PT definition, formula and application are linked to learners' inability to understand the relationship between the hypotenuse and the different sides of the right-angled triangle as reported by the Malawian secondary school teachers. Engagement of learners in establishing relationships is both a language issue and a leaner participation issue (Adler & Alshwaikh, 2019; Prediger, 2022).

Development of mathematical tasks that promote learners' involvement in rich mathematical talk and involvement of learners in such tasks is most challenging for teachers and requires continued PD (Prediger, 2022). While mathematical task development has received much attention in research on language and mathematics, little has been reported on mathematical task implementation (Prediger, 2022). In this paper, I build on the existing research on language and mathematics tasks development and implementation by exploring the teachers' learning about engagement of learners in rich mathematical talk and thinking of PT during LS. Specifically, I pursue the following research questions: 1) how do the geometric tasks developed by the teachers promote learners' involvement in rich mathematical talk? and 2) how does the enactment of the tasks and examples promote learners' involvement in rich mathematical talk?

I start with a brief review of research and arguments for theory-informed LS, some of which has a geometry focus and or is related to language practices. This is followed by an elaboration of the theoretical framing for our LS which lays the ground for the methodology used and the presentation and discussion of the findings.

Research on Theory-informed Lesson Study and/or with a Geometry Content Focus

LS has been adopted and adapted in many countries as an effective and ongoing continuous PD for improving teaching quality, consequently improving learners' learning too (Huang & Shimizu, 2016). LS provides teachers with several opportunities including: 1) learning to improve their teaching practices through practicing planning, teaching and reflecting on their lessons, 2) learning new teaching practices, 3) deepening their subject matter knowledge, and working collaboratively (Huang et al., 2019). Several challenges of adapting LS in new contexts have been reported, in mathematics education including in Malawi (e.g. Ebaeguin & Stephens, 2013; Fujii, 2014; 2016). In African countries, major challenges include time constraints for teachers' which limit them from enacting a typical LS cycle, and teachers' inability to differentiate a normal/usual lesson and a research lesson as they fail to link the lesson to an identified research problem and with mathematical problem solving (Fujii, 2014; 2016; Jita et al, 2008; Sekao & Engelbrecht, 2021).

Fujii's (2014; 2016) reports on Malawian teachers' routine interpretation of mathematical problem solving as mere solving of mathematics problems but not as a process of provoking learners' mathematical thinking and talk during LS. The challenges of teachers' inability to engage learners in correct mathematical problem-solving processes are associated with language challenges in mathematics education and have been globally reported as being rampant and persistent (Adler, 2022; Prediger, 2019). A generally agreed upon solution to this challenge includes engagement of teachers in PD activities that foster their capacity to develop mathematical tasks that provoke learners' rich mathematical discourse (Fujii, 2014; Prediger, 2019). In our previous studies on LS, we argued that designing mathematical tasks that can promote learners' rich mathematical talk and thinking is not trivial for teachers, hence theory-informed frameworks are critical in supporting teachers' development of such capacity during LS (Mwadzaangati et al., 2022; Mwadzaangati, 2024). Therefore, our use of MTF as a tool for lesson planning, observation and reflection has been informed by these studies and our accumulating experience that it can aid teachers' ability to develop tasks that can support rich mathematical talk during LS.

The issues of tasks and language are foregrounded in most of the LS research on the teaching and learning of geometry. Moss et al. (2015) report on an adapted Japanese LS to support early years teachers in the USA in developing content knowledge and new approaches for teaching geometry and spatial reasoning. While they do not explicitly state their theoretical orientations, their mathematical and pedagogical goals were to enable teachers to move from a static and narrow conception of geometry as classification of shapes, to a more dynamic conception, with transformational approaches of reasoning about shapes. The adaptations to the Japanese LS model introduced four additional steps to the typical cycle of planning, teaching and reflecting: (1) teachers engaging in the mathematics in PD workshops, (2) teachers designing and conducting task-based clinical interviews, (3) teachers and researchers co-designing and carrying out exploratory lessons and activities, and (4) the creation of resources for other educators. Each of these adds in considerable time through focused PD work being integrated with the LS cycle. For example, with respect to (1) above, there were spatial reasoning geometry activities for the teachers that included building different types of three-dimensional figures from multi-link cubes and responding to different types of visualisation questions through engaging with two-dimension figure pattern blocks. In our adapted LS model, we also noted that due to the incorporation of PD workshops, time also became an important issue that influenced teachers' learning.

Moss et al. (2015) reported that the adapted LS was effective in broadening the teachers' content knowledge and interest for teaching geometry, and in increasing teachers' perceptions of young children's mathematical competencies. The teachers reported appreciation of and confirmed the need for designing and engaging learners in tasks that provoke their geometric reasoning, with evidence of teachers moving beyond their static views and enactments towards dynamic orientations. Their study also exemplifies adaptations that expand the LS cycle to include direct work on mathematics content with teachers which in this case went further to include co-designing mathematical tasks for promoting learners' mathematical thinking.

Some studies on LS with a geometry content focus that I identified (eg. Bahn & Winsløw, 2019; Chesnais, 2021; Moss et al., 2015) illuminate that bringing mathematical content into focus in LS research and practice where theoretical frameworks for mathematics teaching are at work is not trivial. Even where a mathematical content framework/theory is used, focus on mathematical content might not be a priority for the researchers depending on their interests and research focus. For example, while some studies that responded to the call for use of theory-driven LS in mathematics have used mathematically focused frameworks, their research foci differ. Some have mainly used the theory to research LS, without elaboration of mathematical content learnt by the teachers. For example, in Bahn & Winsløw (2019), geometry and probability questions were used to illustrate how the theory of didactical situations (TDS) can be adapted as a tool for planning and analysing research lessons and learning situations in LS. Research questions focused on the role of different components of LS including how the roles interact, and the effects of repeating research lessons. The teachers' use of the TDS was not detailed as it was not the main focus of the paper. Consequently, the mathematical learning of the teachers was back grounded. A further interesting insight from this study was that despite the LS being oriented to teachers using and talking about TDS through the LS cycle, the teachers did not use TDS-specific language during either planning or post-lesson discussions. Considering that language issues were of explicit interest or concern and focused on during the PD workshops in this study, it was worthwhile to conduct a study focused on these.

The Mathematics Teaching Framework (MTF)

The MTF is the 'teaching' version of the Mathematical Discourse in Instruction analytical framework developed for describing, observing and reflecting on the quality of mathematics made available in teaching (Adler & Alshwaikh, 2019; Adler & Ronda, 2015). MTF draws from key tenets of socio-cultural theory by viewing mathematics as a network of interrelated and well-organised scientific concepts whose teaching and learning is goal directed and accompanied by coherent activities (Adler & Alshwaikh, 2019; Adler & Ronda, 2015; Vygotsky, 1978). As figure 1 shows, the starting point of MTF is that teaching is always about something, called the lesson goal, and the work of mathematics teaching involves bringing into focus the lesson goal also known as mediation. Three key recognisable and inter-connected mediational teaching practices of exemplification, learner participation and explanatory communication work into play to achieve the lesson goal.

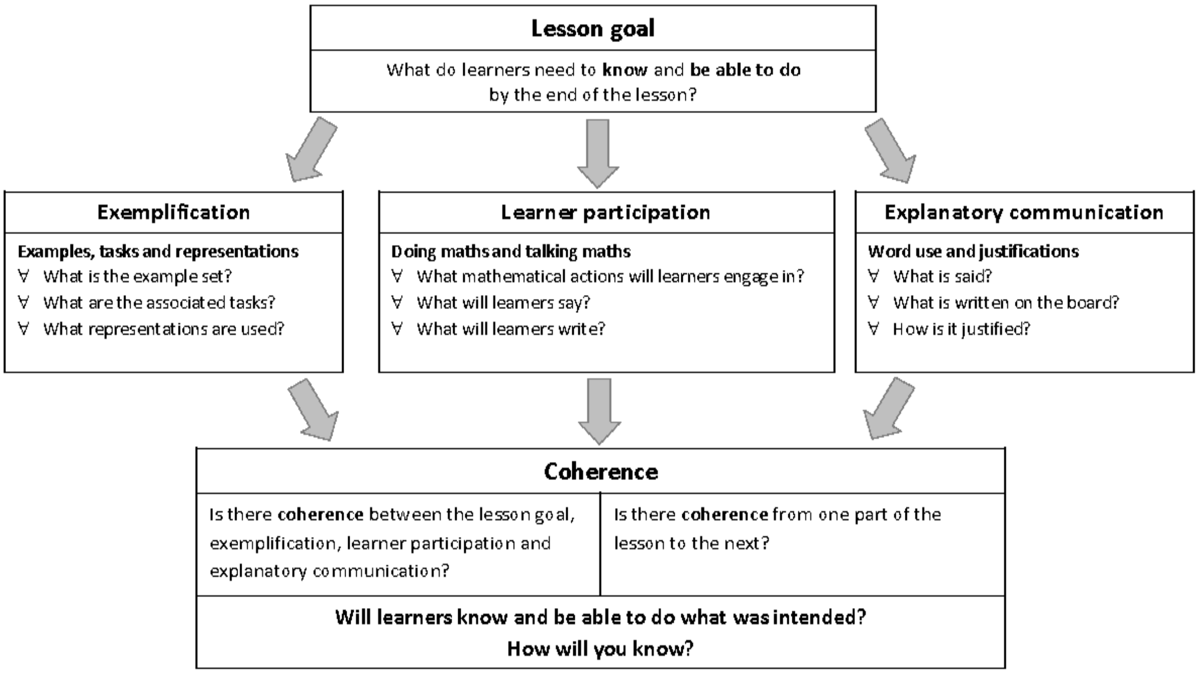

Fig. 1: Mathematics Teaching Framework (Adler & Alshwaikh, 2019)

The related key mediational means for exemplification includes examples, tasks, and their representational forms. Those for explanatory communication include word use and justifying, and those for learner participation include what learners are invited to do, say and write. These mediational means work together to open (or close) opportunities for learners' attainment of the lesson goal. Since the focus of this paper is on teachers' involvement of learners' in rich mathematical talk, then I zoom in on the language practice of explanatory communication and learner participation teaching practices of the MTF.

The MTF suggests that rich mathematical opportunities are made available to learners when they are invited to answer how and why questions than when they answer what and yes/no questions. Attention is also paid on movement between colloquial/everyday, school mathematics and technical/formal mathematical registers. Justification focuses on how words are explained/defined and whether they are elaborated and then in what way. Since MTF links with the work of other scholars working with content specific language practices (e.g. Prediger, 2022), we designed the teachers' activities during the PD of the second cycle of the LS to focus on learner participation and explanatory communication. The aim was to help the teachers to plan and implement lessons that would create opportunities for learners to engage in rich mathematical talk and thinking.

Methodology

The methodology section is divided into 2 phases. The first phase focuses on the process of LS and the second phase focuses on the research itself.

Conducting the lesson study

The PD workshops were planned by four mathematics educators also known as knowledgeable others (KOs) including the author of this paper. Two KOs were from a South African University while the other two were from the Malawian University where the PD workshops took place. In cycle 2 PD workshop we focused on activities for promoting learners' mathematical talk and thinking. We discussed with the teachers how to bring out explanatory communication in terms of word use and justifications/reasoning while promoting learner talk and thinking, a learner participation aspect of MTF. In both cycle 1 and cycle 2, the teachers conducted LS using stages presented in Figure 2.

Each LS cycle consisted of two initial planning sessions (LP1A and LP1B) that were based on addressing the teaching problem identified by the teachers. This was followed by teaching 1, reflection 1 and lesson planning 2, then teaching 2, followed by reflection 2 and lesson planning 3. The author of this paper was present throughout all stages of the LS to video record the proceedings and to offer guidance. We deliberately planned that there should be minimal input from KO during LP1A planning session with an aim of wanting to start from what the teachers know and their take up on what we discussed with them during the PD workshops.

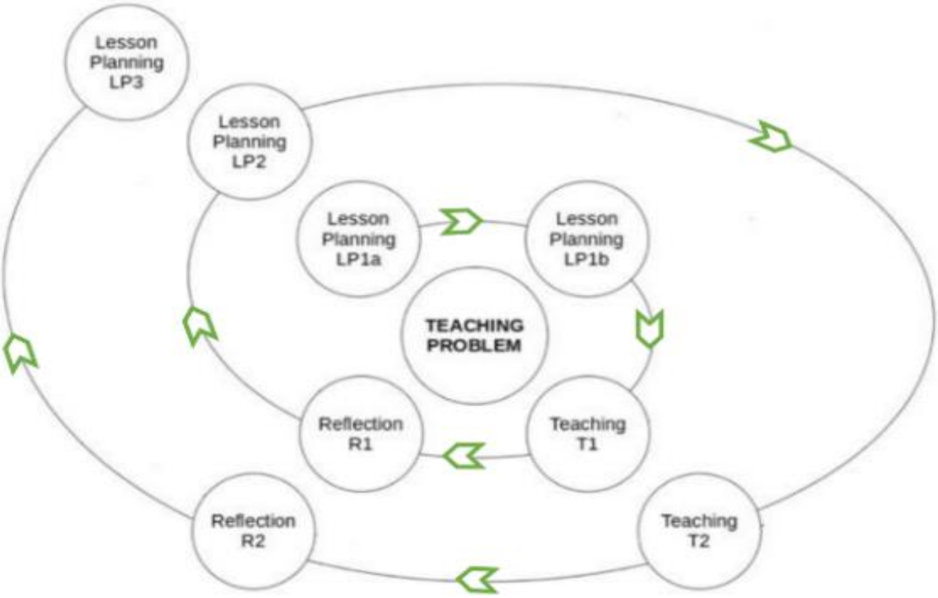

Figure 2: Lesson Study Cycle

Conducting the study

The teachers who were engaged in the LS were purposefully selected. Our plan was to work with teachers who were qualified and experienced to teach secondary school mathematics as we assumed that their content knowledge was much better than the unqualified and inexperienced teachers. We therefore chose teachers whose minimum qualification was bachelor of education degree with mathematics as an area of specialisation and had more than five years of experience in teaching secondary mathematics so that our focus could only be on developing their existing content knowledge and pedagogy. Since these teachers were teaching at a national secondary school, it followed that the learners were of higher abilities than those of district and community day secondary schools. Out of the five teachers being focused on in this study, one was female teacher and four were male teachers. Each class comprised 40 learners.

For this study, the main data are the teachers' two lesson plans and two lessons delivered using the lesson plans. The data was analysed qualitatively starting with transcription of the video recorded lessons, followed by reading the transcribed data several times and dividing it into lesson episodes related to different lesson foci. The lesson transcriptions capture line by line utterances together with illustrations of teacher and learner activity in the classrooms. Further data analysis involved coding the data using codes that I developed from the MTF. To find out whether and how learners' tasks promoted learners' involvement in rich mathematical talk, I analysed the learners' activities in the LPs to examine the type of questions and explanatory communication asked or inferred. To find out whether and how the enactment of the learners' tasks promoted learners' involvement in rich mathematical talk, I analysed the two lessons to examine how learners were involved in word use and justification, including the type of questions they were invited to respond to when carrying out or reporting their tasks. Regarding coding of the data, on learner participation, where what questions were used/inferred, I coded QWhat, where how or why questions were asked/inferred, I coded QHow and QWhy respectively. On explanatory communication, where talk only included naming without justification I coded JNA, if justification only included stating the definition or reason (JNM), or if it also included elaborating the definition or reason (JEM). In the following section I present the findings of the study.

Compliance with ethical considerations

Several ethical considerations were considered considering that data for different studies was to be generated at each phase of the LS, (Bryman, 2008). We started by seeking permission from the Education Divisional Office to gain access into the schools. The Education Division Manager granted us permission into the schools and an introductory letter to the head teachers of the schools. To collect data from the teachers and observe lessons, we sought verbal informed consent from each teacher upon explaining to them the purpose of the LS and how we were going to collect and treat the data. Before each lesson, we sought consent from the learners by informing them that we were conducting a study and asking them if we could video record the lessons. Both the teachers and learners were assured of anonymity and confidentiality, and this is evidenced through the use of pseudonyms in reporting the findings.

Findings

In both lesson plans (LPs) and lessons, the focused object of learning is the meaning of hypotenuse and PT. In both lesson plans and lessons, the teachers ensure that they follow the common sequence of starting with the introduction where they recall knowledge of types of triangles. Then in the lesson development stage, they ask learners to do activities for inducing properties of a hypotenuse and PT. This is followed by doing examples for illustrating application of PT. In their conclusion, they ask learners to explain what they have learnt about the hypotenuse and PT. In this paper I focus on episodes 2 and 3 where the emphasis was on the issue that teachers wanted to help their learners understand, thus the hypotenuse. In these episodes, learners were involved in exploring and inducing the hypotenuse and its relation to the other two sides of the right-angled triangle and the PT. I begin by presenting findings on types of tasks and questions that were asked/inferred in the LPs and the type of mathematical talk that would be expected from these, followed by findings on the mathematical talk that was promoted during the lessons.

Table 1: LP tasks for developing understanding of hypotenuse and Pythagoras Theorem

Teaching 1	**Teaching 2**
Tasks for Lesson episode 2: developing meaning of hypotenuse ● Measure the lengths of the given triangles and indicate them on the triangles. **Qwhat, JNA** ● Identify the longest side in the given triangles. **Qwhat, JNA** ● Identify the hypotenuse in the given triangles. **Qwhat, JNA** ● Identify sides known as legs in the given triangles. **Qwhat, JNA** ● Define hypotenuse in their own words. **Qwhat, Qwhy, JNM, JEM** **Tasks for lesson episode 3.1: developing relationship of the areas of squares** ● Draw squares on each side of the triangle they were using in activity 1. ● Find areas of the drawn square using the measurements they found in activity 1. **Qwhat, Qhow, JNA, JEM** ● Observe the relationship of the areas in each triangle and fill in the given boxes. **Qwhat, Qhow, JNA, JEM** ● Explain what they notice if they add the areas of the squares on the legs and compare it with the area of the hypotenuse **Qwhat, Qwhy, JEM** **Tasks for lesson episode 3.2: developing understanding of the Pythagoras Theorem** ● Replace numbers they wrote on the sides of the triangles in the given activity sheet with letters such as **a**, **b**, and **c**. **Qwhat, JNA** ● Find areas of the squares in terms of letters. **Qwhat, Qhow, Qwhy, JNA** ● Give the relationship of the areas of the squares in terms of letters used. **Qwhat, Qhow, Qwhy, JEM**	**Tasks for lesson episode 2: developing meaning of hypotenuse** ● Measure the lengths of the given triangles and indicate them on the triangles. **Qwhat, JNA** ● Explain what you notice about the side opposite 90° angle in the given triangles. **Qwhat, Qwhy, JEM** ● Give the name of the side opposite 90° angle. **Qwhat, JNA** ● State the other name given to the other two sides of a right-angled triangle. **Qwhat, JNA** ● Ask learners to define hypotenuse in their own words. **Qwhat, Qwhy, JNM, JEM** **Tasks for lesson episode 3.1: developing relationship of the areas of squares** ● Find areas of the squares drawn on the sides of the right-angled triangles given. **Qwhat, Qhow, JNA, JEM** ● Observe the relationship of the areas of the squares in each triangle and fill in the given boxes. **Qwhat, Qhow, JNA, JEM** ● Explain what they notice if they add the areas of the squares on the legs and compare it with the area of the square on the hypotenuse. **Qwhat, Qhow, JNA, JEM** **Tasks for lesson episode 3.2: developing understanding of the Pythagoras Theorem** ● Use letters **a**, **b** and **c** instead of numbers they wrote on the sides of the triangles in the given activity sheet. **Qwhat, JNA** ● Find areas of the squares in terms of letters and fill in the boxes aside. **Qwhat, Qhow, Qwhy, JNA** ● Give the relationship of the areas of the squares in terms of letters used. **Qwhat, Qhow, Qwhy, JNA, JNM, JEM**

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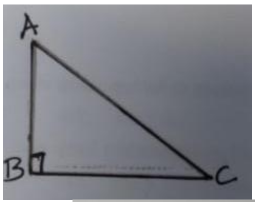
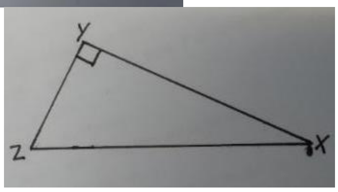
- State the relationship of the areas of the squares of the right-angled triangle using their own words. **Qwhat, Qhow, Qwhy, JEM**

Diagrams for the tasks





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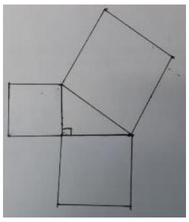
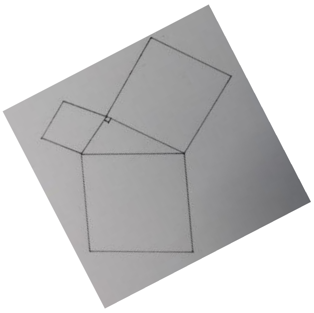
- State the relationship of the areas of the squares of the right-angled triangle in their own words. **Qwhat, Qhow, Qwhy, JNA, JNM, JEM**

Diagrams for the tasks

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From the learners' tasks in the LPs for both teaching 1 and 2, it can be inferred that learner participation would involve answering both non-rich mathematical talk questions coded Qwhat, and rich mathematical talk questions coded Qhow and Qwhy. What questions are inferred from tasks such as 'measure the lengths of the given triangles and indicate them on the triangles' as the teacher might only ask learners to name the measure of the length after the measurement. How questions are inferred from tasks such as 'find areas of the drawn square using the measurements they found in activity', as the teacher might not only ask the learners to name the measure of the area but also the procedure, (how they found the areas). Why questions can be inferred from tasks like 'define the hypotenuse' as the teacher might ask the learners to both explain the meaning of hypotenuse as well as either why it is called the hypotenuse or why it is the longest side when compared to the other two sides. It is assumed that when the teacher asks the how and why questions in addition to the what questions, then learners would be engaged in rich mathematical talk of making connections between the sides and the angles.

Explanatory communication would involve naming words coded JNA, naming meanings only, coded JNM and elaborating meanings coded JEM. Specifically, in both teaching 1 and 2, lesson episodes 2 on developing understanding of the hypotenuse, learners mathematical talk would involve naming (JNA) the longest side, hypotenuse and legs, as well as rich mathematical talk of justifying by naming and elaborating the definition of hypotenuse (JNM, JEM). It can therefore be implied that learners will be involved in rich mathematical talk during answering how and why questions, when given opportunities to probe for meaning elaboration on the definition of hypotenuse. Similarly, in both teaching 1 and 2 tasks in the LPs, in lesson episode 3.1 for developing understanding of PT, learners' mathematical talk would involve naming the measures of the areas (JNA) and rich mathematical talk of explaining the relationship/meaning of the areas (JEM). In general, it can be inferred therefore that in both lessons, learners would be involved in answering what, how and why questions, hence engaged in rich mathematical talk if the teacher probes for meaning elaboration and reasoning.

Enactment of the learners' tasks for developing understanding of the hypotenuse and the Pythagoras Theorem.

In both lessons, after recalling prior knowledge which included naming of different types of triangles, defining right-angled triangle and drawing on the chalk board different types of right-angled triangles in lesson episode 1, the teachers aimed at getting learners involved in exploring and inducing the meaning of hypotenuse and PT through measuring of the sides and using area model. This was the new thing that they agreed to do during lesson planning as it is different from their routine practice which they said was to first present the theorem using a diagram and equation, then start applying it using examples. Therefore, the lesson proceeded with an empirical activity for exploring the properties of a hypotenuse in a right-angled triangle (lesson episode 2). During the lesson, the teacher gave learners in groups worksheets containing diagrams indicated in Table 1 and asked them to do tasks 2 and 3. In teaching 2, the teacher asked the learners to label the triangles as ABC and XYZ like those for teaching 1. When the learners were reporting, the teacher wrote on the board the measures (board work not shown because it is not being focused on). All groups of learners reported the same measures; the measurements of triangle ABC were AB= 3M, BC =4 CM and AC = 5CM, and the measurements for triangle XYX were ZY= 5CM, XY = 12CM and XZ =13CM. The learners identified the longest side and hypotenuse as AC and the legs as AB and BC in triangle ABC, and in triangles XYZ, they identified XZ as the longest side and hypotenuse and XY and YZ as the legs. The discussion on definition of hypotenuse was as in the following extract from lesson episode 2.

The results on implementation of tasks for exploring the hypotenuse and inducing its property which I do not show in this paper due to space limitations indicated that learner participation involved answering what questions only while explanatory communication only involved naming (JNA) for example the longest side and hypotenuse, as well as naming meaning of hypotenuse (JNM). Elaboration of the meaning of hypotenuse (JEM) was done by the teacher in both teaching 1 and 2. Similar findings are observed in the lesson episodes for developing understanding of relationships of areas of squares and the PT as shown in Table 2. Note that T is for teacher, L for learner and ML for many learners.

Table 2: Lesson Episode 2 exploring and inducing the Pythagoras Theorem

Teaching 1	**Teaching 2**
Lesson episode 3.1: developing relationship of areas.	**Lesson episode 3.1: developing relationship of areas.**
344.T. So, what do you notice when you add the areas of the legs and compare it with the area of the hypotenuse. **Qwhat**	**270.** **T:** Now, what do you notice if you add the squares and compare them with the area of the squares on the hypotenuse? **Qwhat**
345. **L:** If you square the legs and add it will give us the hypotenuse squared. **JNM**	**271.** **L:** The sum of the squares of the legs is equal to the square of the hypotenuse. **JNM**
346.T: (Teacher writes on chalkboard the learner's response). Can we improve that one? **Qwhat** **347.** **L:** The area of 2 legs is equal to the area of the hypotenuse **JNM**	**272.** **T:** (Teacher writes on chalkboard the learner's response). The sum of the squares of the legs is equal to the square of the hypotenuse. So, that's the relationship. Can you write that relationship somewhere

348. **T:** (Teacher writes on chalkboard the learner's response). Okay let us look at key words, from whatever we are doing, in both of these there is addition, we are adding somewhere, right? **Qwhat, JEM**

349. **ML:** Yes.

350. **T**: Somewhere we have squared not so? (referring to learners' task pasted on board) **Qwhat, JEM**

351. **ML:** Yes.

354. **T:** Is addition happening? **Qwhat**

355. **ML:** Yes.

356. **T:** Someone is saying area, are we working with areas? **Qwhat, JEM**

357. **ML**: Yes.

358. **T:** Okay so we can combine the 2 definitions and come up with something. You can see that the square of the hypotenuse is equal to the sum of the squares of the 2 remaining sides. So the square of the hypotenuse is equal to the sum of the squares of the 2 remaining sides. So, take note of that. **JEM**

Lesson episode 3.2: developing understanding of Pythagoras Theorem.

379. **T:** Now I want you to use letters instead of numbers. What will the relationship be like if we use letters of the right angled triangles a, b, c or x, y, z? **Qwhat**

380. **L**: a squared plus b squared is equal to c squared. JNM

381. **T:** a squared plus b squared is equal to c squared. What is c squared? **Qwhat**

382. **ML:** Hypotenuse. **JNM**

383. **T**: Hypotenuse

384. **T:** What about a squared? **Qwhat**

386. **ML:** a. **JNM**

on a paper? (after 1 minute). Have we written that?

275. **ML**: Yes!

Lesson episode 3.2: developing understanding of Pythagoras Theorem.

276. **T:** Okay, we now want to use letters. Use the letters a, b and c instead of numbers. We have written numbers there if you choose triangle second triangle letters x y z (after 3 minutes) what we have you found? **Qwhat**

277. **L:** a squared plus b squared equals c squared

304. **T:** Aright, now the relationship existing between the squares using the letters is that a squared plus b squared is equal to c squared. Can you give me in your own words this relationship? **Qwhat**

305. **L:** The sum of areas of legs is the same as the area of hypotenuse. **JNM**

306. **T**: He is saying the sum of areas of the legs is the same as the areas of the hypotenuse. Any other, yes! **Qwhat**

310. **L:** The sum of the squares of a and b is equal to the square of c. **JNM**

. **T**: The sum of the squares of a and b is equal to the square of c. Let's find out about these relationships. The sum of the areas of the legs is the same as the areas of the hypotenuse. Is this true? And the sum of the squares of a and b is equal to the square of c. Is this also true? **Qwhat**

315. **ML**: Yes!

316. **T**: Now, I want us to polish something here, mathematically. How can we say that the sum of

389.	**T**: Others are saying a, that one is a leg, what about b squared? **Qwhat, JEM**		areas of legs is the same as. What word can we put to replace the same as. **Qwhat JEM**
390.	**ML**: b.	**317.**	**L**: Is equal to. **JNM**
391.	**T:** b is a leg. So, I'm expecting you to say something in terms of those words. **Qwhat**	**319.**	**T**: Okay, this is called the Pythagoras Theorem, the sum of the square of the legs is equal to the square of the hypotenuse and is only applicable in a right-angled triangle. **JEM**
392.	**l:** The sum of 2 squares of legs is equal to hypotenuse. **JNM**		
392: T:	Any other suggestions? **Qwhat**		
400.	**S:** The sum of 2 squares of legs is equal to hypotenuse squared. **JNM**		
409.	**T**: So, Pythagoras theorem states that in a right-angled triangle, the area of the square on the hypotenuse is equal to the sum of areas on squares on the other 2 sides. Please take note that this theorem only applies to right angled triangles. **JEM**		

As Table 2 has shown, in both teaching 1, learners only answer what questions (Qwhat). Although the teachers try to probe or ask follow up questions, they do so, using what questions as well (eg. 348-357). The learners' mathematical talk included naming sides as well as symbols that represent sides (JNM) for example hypotenuse, a and b (386, 390). Justification by learners only involves naming meanings (JNM), for example 'a squared plus b squared is equal to c squared' and 'the sum of 2 squares of legs is equal to hypotenuse squared' (380, 392). Elaboration of the meanings of the relationships is done by the teacher, for example, he elaborates the meaning (JEM) of the relationship of the squares drawn using sides of the right-angled triangles in lesson episode 3.1 (348-358) and the meaning of PT in lesson episode 3.2 (409). He however attempted to engage with learners' thinking by analysing and relating the learners' responses, "if you square the legs and add it will give us the hypotenuse squared (347) and "the area of 2 legs is equal to the area of the hypotenuse (349) to come up with the PT "the square of the hypotenuse is equal to the sum of the squares of the 2 remaining sides" (409).

During lesson reflection, one issue that emerged as a great challenge for the teachers was that learners spent a long-time drawing squares on each side of the right-angled triangles indicated in Table 1 LP for teaching 1. As such the teachers agreed to draw the right-angled triangles in advance as shown in the LP for teaching 2 so that learners would only calculate areas. However, the teachers were happy with the way their fellow teacher engaged learners in mathematical talk. But after we watched the Teaching 1 as KOs, we noticed that although meaning elaboration was emphasised, it was mainly done by the teacher and not the learners. Therefore, during the second PD workshop which was initial planning for teaching 2, we engaged the teachers in a variety of activities aimed at supporting their capacity to engage learners in more and rich mathematical talk. The teachers also watched and critically analysed an exemplary video lesson where more and rich mathematical talk was promoted. Although the teachers agreed to the need for engaging learners in meaning elaboration upon watching recording of their teaching 1 and comparing it to the exemplary video lesson that they watched, as Table 2 has shown, still more, there was little focus on meaning elaboration in Teaching 2.

As the findings show in Table 2, in teaching 2, learners were also only engaged in answering what questions (270, 276, 306), and naming meanings (271, 307). When one learner mentioned the relationship of the areas, "the sum of the squares of the legs is equal to the square of the hypotenuse" the teacher did not relate this to what learners wrote on the board, he only asked them if they agreed with the response (271, 280). This was different from Teaching 1 where the teacher attempted to relate the learners' explanation and the measures they found. He however, ensured correct naming of an arithmetic operation when he asked learners to replace 'same as' with a correct mathematical word (306). After inducing and elaborating the PT, both teachers emphasised that it only applies to right-angled triangles (409, 319), this is what they agreed to do to help learners understand how to apply the theorem. Thus, in general, the findings imply that while the teachers understand the need for engaging learners in rich mathematical talk to help them to understand the hypotenuse and PT, bringing this talk into focus remained a challenge even after completing the LS cycle.

Study Limitations

This study has some limitations as it is the case with small qualitative studies. Firstly, I cannot claim that the teachers' language practices continued to improve after the LS unless a follow-up study is conducted. Secondly, the findings of the study do not illuminate whether the improvement in teachers' language practices really resulted in improvement in learners' understanding of the concept of PT because the focus of the study was only on the teachers but not learners. However, what is evident is that LS does create opportunities for teachers' learning about improving their mathematical language practices.

Discussion and Conclusion

In this paper, I explored whether and how tasks planned by teachers as well as their implementation promote learners' engagement in deep mathematical talk during LS on PT. The teachers were exploring how they could help learners understand PT and the difference between the hypotenuse and the other sides of a right-angled triangle, which was a challenge for learners when applying PT to calculate geometry problems. They agreed to present the lessons on PT differently through engaging learners in rich mathematical talk while exploring and inducing the hypotenuse and the PT. As the findings have shown, the tasks planned by the teachers had potential of engaging learners in rich mathematical talk because they were to some extent open ended as suggested by Prediger (2022). Since the tasks could enable the teachers to ask how and why questions, they could afford probing for meaning elaboration and justification (Adler & Ronda, 2015). But as the findings have shown, in both lessons, the teachers only asked what questions without following up with how or why questions during both lessons. This led to engagement of learners in low level language practices of naming words and meanings only, but not in high level language practice of meaning elaboration as is suggested in literature (Adler & Ronda, 2015; Adler, et al., 2022; Prediger, 2022). The findings confirm general literature on mathematical tasks that although teachers or textbooks can be capable of setting up high cognitive demand tasks that can engage learners in high cognitive mathematics learning, the implementation of the tasks matters most as it can either maintain or demote the cognitive value of such tasks (Mwadzaangati, 2019; Stein et al., 2009).

Teacher knowledge is one of the factors that influence engagement of learners in rich mathematical talk, hence the requirement for engaging teachers in PD to promote such skills is emphasised in research Mwadzaangati, 2024; Prediger, 2022). During the PD, the teachers appreciated the need and importance of involving learners in rich mathematical talk to help them understand the meaning of PT and the difference between the hypotenuse and the other two sides of the right-angled triangle. To this effect, they agreed to engage learners in empirical exploration through several tasks instead of their routine abstract way of teaching the theorem. This is evidenced in their focusing on elaborating meanings and the use of two right-angled triangle diagrams to promote talk through comparison. However, the fact that teachers grappled with engagement of learners in elaborating meanings implies that language responsive teaching remains a challenging practice and confirms the need for engagement of teachers in continuous PDs (Moss et al., 2015; Prediger, 2019). This means that although LS is a powerful PD for improving teaching practice, it is difficult for teachers to understand all practices of language and mathematics in a single PD and LS cycle and implement them successfully. The difficulties might be more challenging in contexts where LS is a new practice, as teachers are not only learning how to engage learners in rich mathematical talk, but also how to participate in LS.

I conclude that while studying through LS opens up opportunities for teachers to learn about improving their language practice, development of the language practice is a long-term goal that requires providing both in-service and pre-service teachers with more opportunities for learning how to support learners' rich mathematical talk. This calls for reconceptualization of LS with PDs for mathematics teachers as a regular PD for building teacher's capacity in different issues in mathematics education including language responsive teaching.

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